Maxwell- Boltzmann Distribution

(Post Graduate Level)

Paper Code-MPHYCC-8

By

Dr. Akhilesh Kumar Gupta

Assistant Professor

Department of Physics

Patna Science College

Patna University, Patna

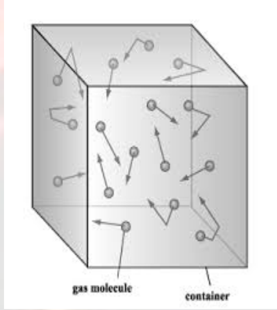
Maxwell-Boltzmann Distribution Law

Maxwell-Boltzmann statistics is classical statistics, which is given for the classical particles.
Following are the basic postulates of MB statistics:

- The associated particles are distinguishable.
- Each energy state can contain any number of particles.
- Total number of particles in the entire system is constant.
- Total energy of all the particles in the entire system is constant.
- Particles are spinless. Example: gas molecules at high temperature and low pressure.

Classical Particles: Classical particles are identical but far enough to be distinguishable. The wave functions of the classical particles do not overlap on each other.

Distinguishable: Two particles are said to be distinguishable if their separation is large in compare to their De-Broglie wavelength. For distinguishable particles you would know if two particles changes their places.

E=Total energy of the entire system = Constant. N=Total number of identical distinguishable particles=Constant V= Total volume =Constant	

Dr. Akhilesh Kumar Gupta, Assistant Professor, Department of Physics, Patna Science College, Patna University, Patna Email: akhil.inphy@gmail.com Contact: +91-8789435605

We now focus on the number of particles siting in given energy levels ε_1 , ε_2 , ε_3 .... ε_n which are available within the system. The energy levels are fixed for the system.

The number of particles in each energy levels are variable and given by n_1 , n_2 , n_3 .... n_r .

The number of ways to attained a given microscopic state is given by

$$\omega = \frac{N!}{n_1!n_2!\ldots\ldots\ldots\ldots n_r!} \qquad \text{--------------(1)}$$

We need to know the distribution of the particles in different energy levels (as stated above) that maximize the value of **ω.**

The combination result in most probable microstate and in this most probable state the system is considered as the equilibrium state.

Now,

$$\log \omega = \ln \frac{n!}{\Pi_{i=1}^{i=r} n_i!} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

For maximum value of ω instead of dealing with ω deal with logarithmic of ω .

$$\log \omega = \log N! - \sum_{i=1}^{i=r} \ln n_i! \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Using Stirling's approximation

$$\log x! \approx x\log x - x \qquad \text{....................(4)}$$

Equation (3) can be expressed using above approximation as

$$\log\omega = N\log N - \sum_{i=1}^{i=r}\left[n_i \ln n_i - n_i\right]$$

Taking the derivative of the above equation,

$$\delta\log\omega = -\sum_{i=1}^{i=r}\delta n_i \ln n_i + n_i \times \tfrac{1}{n_i} - \delta n_i = 0$$

$$\sum_{i=1}^{i=r}\delta n_i \ln n_i = 0 \qquad \text{--------------(5)}$$

$$\sum n_i = N = \text{Constant}$$

$$\sum \delta n_i = 0 \qquad \text{---------------(6)}$$

(The sum of all the changes in the system is zero)

Total energy,

$$E = \sum \varepsilon_i n_i = \text{Constant}$$

On differentiating the above equation

$$\sum_{i=1}^{i=r}\varepsilon_i \delta n_i = 0 \qquad \text{----------------(7)}$$

Dr. Akhilesh Kumar Gupta, Assistant Professor, Department of Physics, Patna Science College, Patna University, Patna Email: akhil.inphy@gmail.com Contact: +91-8789435605

To maximize the function in (5) subjected to constrain (6) and (7) let us use Lagrange's method of undetermined multipliers equation (6) multiplied by α and equation (7) is multiplied by β and the adding equation (5) , (6) and (7) we get,

$$\sum\left[\ln n_i + \alpha + \beta\varepsilon_i\right]\delta n_i = 0$$ ------------(8)

$$\sum\left[\ln n_i + \alpha + \beta\varepsilon_i\right] = 0$$

$$\ln n_i = -\alpha - \beta\varepsilon_i$$

$$n_i = e^{-\alpha} . e^{-\beta\varepsilon_i}$$ ---------------(9)

=The number of particle in i^{th} level

$$\sum_{i=1}^{i=r} n_i = e^{-\alpha}\sum_{i=1}^{i=r} e^{-\beta\varepsilon_i} = N$$

$$e^{-\alpha} = \frac{N}{\sum_{i=1}^{i=r} e^{-\beta\varepsilon_i}}$$ ……………..(10)

$$e^{-\alpha} = \frac{N}{p}$$

where,

$$p = \sum_{i=1}^{i=r} e^{-\beta\varepsilon_i}$$

$$n_i = \frac{N}{p} e^{-\beta\varepsilon_i}$$ ....(11)

Dr. Akhilesh Kumar Gupta, Assistant Professor, Department of Physics, Patna Science College, Patna University, Patna Email: akhil.inphy@gmail.com Contact: +91-8789435605

$$\beta = \frac{1}{k_\beta T}$$

$$n_i = \frac{N}{p} e^{\frac{-\varepsilon_i}{k_\beta T}} \qquad \text{------------(12)}$$

The above expression helps us to determine the number of particles in most probable micro-states. The expression is known as Maxwell- Boltzmann statistics expression.

The probability of a particle to occupy the energy state ε_i is given by Maxwell-Boltzmann function

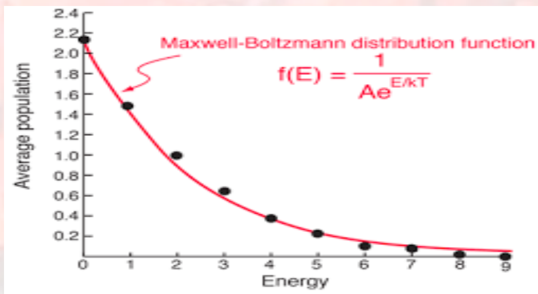
$$f(\varepsilon_i) = \frac{n_i}{g_i} = \frac{1}{e^{\alpha + \beta\varepsilon_i}} \qquad \text{----------------(13)}$$

$$f_{MB}(\varepsilon) = Ae^{-\varepsilon/k_\beta T} \qquad \text{-----------------(14)}$$

Where,

A=Constant

A depends on the number of particles in the system and plays the same role like normalization constant.

k_β = Boltzmann Constant = 1.38×10^{-23} J/K =8.617×10^{-3} eV/K g_i = Number of quantum states of the i^{th} energy level	*Graphical representation of Maxwell-Boltzmann Distribution* 

Dr. Akhilesh Kumar Gupta, Assistant Professor, Department of Physics, Patna Science College, Patna University, Patna Email: akhil.inphy@gmail.com Contact: +91-8789435605